

Solitons in the Calogero model for distinguishable particles

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Abstract

We consider a large $-N$, two-family Calogero model in the Hamiltonian, collective-field approach. The Bogomol'nyi limit appears and the corresponding solutions are given by the static-soliton configurations. Solitons from different families are localized at the same place. They behave like a paired hole and lump on the top of the uniform vacuum condensates, depending on the values of the coupling strengths. When the number of particles in the first family is much larger than that of the second family, the hole solution goes to the vortex profile already found in the one-family Calogero model.

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The Calogero system is a class of exactly solvable models in one dimension [1-3]. The model has found wide applications in areas as diverse as condensed matter [4], black-hole physics [5] and two-dimensional string theory [6]. The ordinary Calogero model describes N indistinguishable particles on the line which interact through an inverse-square two-body interaction. As far as the distinguishable particles are concerned there are a few generalizations of the Calogero model to models of particles with different masses and with $\frac{1}{r_{ij}^2}$ couplings depending on the labelling of the particles coupled [7-14]. A multispecies one-dimensional Calogero model with two- and three-body interactions was treated in the $SU(1,1)$ algebraic approach in [12, 13], while its matrix formulation was presented in [14]. Although it was possible to find an infinite number of exact eigenstates and eigenenergies, the set is not complete.

Recently, it was shown in Ref. [15] that a natural supersymmetric extension of the Calogero model resulted in two-family Calogero models which are exactly solvable in some special cases.

In an attempt to better understand the nature of the multispecies Calogero model, in this paper we transform the two-family model to collective fields. This transformation gives much more insight into the non-perturbative, solitonic sector of the theory.

Let us start with the Calogero Hamiltonian describing two different families of particles in interaction [13]:

$$H = -\frac{1}{2m_1} \sum_i^{N_1} \frac{\partial^2}{\partial x_i^2} + \frac{\lambda(\lambda-1)}{2m_1} \sum_{i \neq j}^{N_1} \frac{1}{(x_i - x_j)^2} - \frac{1}{2m_2} \sum_{\alpha}^{N_2} \frac{\partial^2}{\partial x_{\alpha}^2} + \frac{\nu(\nu-1)}{2m_2} \sum_{\alpha \neq \beta}^{N_2} \frac{1}{(x_{\alpha} - x_{\beta})^2} \\ + \frac{1}{2} \sum_i^{N_1} \sum_{\alpha}^{N_2} \frac{\kappa(\kappa-1)}{(x_i - x_{\alpha})^2} \left(\frac{1}{m_1} + \frac{1}{m_2} \right) +$$

$$\begin{aligned}
& + \frac{1}{2} \sum_i^{N_1} \sum_{\alpha \neq \beta}^{N_2} \left(\frac{\kappa^2}{m_1(x_i - x_\alpha)(x_i - x_\beta)} \right) + \sum_i^{N_1} \sum_{\alpha \neq \beta}^{N_2} \left(\frac{\nu \kappa}{m_2(x_\alpha - x_i)(x_\alpha - x_\beta)} \right) \\
& + \frac{1}{2} \sum_{i \neq j}^{N_1} \sum_{\alpha}^{N_2} \left(\frac{\kappa^2}{m_2(x_\alpha - x_i)(x_\alpha - x_j)} \right) + \sum_{i \neq j}^{N_1} \sum_{\alpha}^{N_2} \left(\frac{\lambda \kappa}{m_1(x_i - x_\alpha)(x_i - x_j)} \right). \quad (1)
\end{aligned}$$

The first family contains N_1 particles with mass m_1 at positions x_i , $i = 1, \dots, N_1$, while the second one contains N_2 particles with mass m_2 at positions x_α , $\alpha = 1, \dots, N_2$. The particles of the same kind interact and the corresponding coupling constants within each family are given by λ and ν , respectively. The particles of different kind also interact and the interaction strength between the first and the second family is denoted by κ . We consider the parameters λ, ν and κ positive.

The Hamiltonian (1) describes the simplest multispecies Calogero model for particles on the line, interacting with the two- and three-body potentials. Setting $\lambda = \nu = \kappa$ and $m_1 = m_2$, we recover the ordinary N -body Calogero model. The three-body terms in (1) trivially vanish in this case. General conditions for the absence of three-body interactions are given in [12, 13]. In the following we do not use any confining potentials.

We can perform the similarity transformation

$$H \rightarrow \Pi_\kappa^{-1} \Pi_\nu^{-1} \Pi_\lambda^{-1} H \Pi_\lambda \Pi_\nu \Pi_\kappa \quad (2)$$

to obtain a simpler but non-hermitian Hamiltonian

$$\begin{aligned}
H = & -\frac{1}{2m_1} \sum_i^{N_1} \frac{\partial^2}{\partial x_i^2} - \frac{1}{m_1} \left(\lambda \sum_{i \neq j}^{N_1} \frac{1}{x_i - x_j} + \kappa \sum_{i, \alpha} \frac{1}{x_i - x_\alpha} \right) \frac{\partial}{\partial x_i} \\
& -\frac{1}{2m_2} \sum_{\alpha}^{N_2} \frac{\partial^2}{\partial x_\alpha^2} - \frac{1}{m_2} \left(\nu \sum_{\alpha \neq \beta}^{N_2} \frac{1}{x_\alpha - x_\beta} + \kappa \sum_{i, \alpha} \frac{1}{x_\alpha - x_i} \right) \frac{\partial}{\partial x_\alpha}, \quad (3)
\end{aligned}$$

where the two- and three-body interactions have simply disappeared. The Jastrow prefactors are given by

$$\begin{aligned}\Pi_\lambda &= \prod_{i < j}^{N_1} (x_i - x_j)^\lambda, \\ \Pi_\nu &= \prod_{\alpha < \beta}^{N_2} (x_\alpha - x_\beta)^\nu, \\ \Pi_\kappa &= \prod_{i, \alpha}^{N_1 N_2} (x_i - x_\alpha)^\kappa\end{aligned}\tag{4}$$

and incorporate the conditions that the wave functions go to zero whenever the particles approach each other. The usual approach to the quantum-mechanical problem is to solve the eigenvalue problem with the Hamiltonian (1) using symmetric and antisymmetric wave functions, depending on the underlying statistics of the identical particles. Instead, we develop a collective-field theory of this system in the large $-N_1$ and large $-N_2$ sectors of the Hilbert space.

The collective-field theory for the two-family Calogero model is obtained by changing variables from the particle coordinates x_i and x_α to the density fields $\rho(x)$ and $\tilde{\rho}(x)$ defined as

$$\rho(x) = \sum_{i=1}^{N_1} \delta(x - x_i),\tag{5}$$

$$\tilde{\rho}(x) = \sum_{\alpha=1}^{N_2} \delta(x - x_\alpha).\tag{6}$$

Such a change of variables is meaningful only if the particle numbers N_1 and N_2 go to infinity [16, 17, 18]. The Hamiltonian (3) can be expressed entirely in terms of $\rho(x)$, $\tilde{\rho}(x)$ and their canonical conjugates

$$\pi(x) = -i \frac{\delta}{\delta \rho(x)},\tag{7}$$

$$\tilde{\pi}(x) = -i \frac{\delta}{\delta \tilde{\rho}(x)},\tag{8}$$

satisfying the following equal-time commutation relations:

$$[\rho(x), \pi(y)] = i\delta(x - y), \quad (9)$$

$$[\tilde{\rho}(x), \tilde{\pi}(y)] = i\delta(x - y), \quad (10)$$

$$[\rho(x), \tilde{\rho}(y)] = [\pi(x), \tilde{\pi}(y)] = 0. \quad (11)$$

After the change of the variables, the Hamiltonian (3) takes the form

$$\begin{aligned} H = & \frac{1}{2m_1} \int dx \rho(x) (\partial_x \pi(x))^2 \\ & - \frac{i}{m_1} \int dx \rho(x) \left(\frac{\lambda - 1}{2} \frac{\partial_x \rho}{\rho} + \lambda \oint \frac{dy \rho(y)}{x - y} + \kappa \oint \frac{dy \tilde{\rho}(y)}{x - y} \right) \partial_x \pi(x) \\ & + \frac{1}{2m_2} \int dx \tilde{\rho}(x) (\partial_x \tilde{\pi}(x))^2 \\ & - \frac{i}{m_2} \int dx \tilde{\rho}(x) \left(\frac{\nu - 1}{2} \frac{\partial_x \tilde{\rho}}{\tilde{\rho}} + \nu \oint \frac{dy \tilde{\rho}(y)}{x - y} + \kappa \oint \frac{dy \rho(y)}{x - y} \right) \partial_x \tilde{\pi}(x), \end{aligned} \quad (12)$$

where \oint denotes Cauchy's principal value of the integral. This Hamiltonian is still non-hermitian owing to the imaginary terms. In order to obtain the hermitian Hamiltonian, we have to rescale Schrodinger's wave functions of the original Hamiltonian by using the Jacobian of the transformation from $\{x_i, x_\alpha\}$ to $\{\rho(x), \tilde{\rho}(x)\}$, as was suggested in Ref. [16]. After performing a straightforward algebra, we find the Jacobian J

$$\begin{aligned} \ln J = & (1 - \lambda) \int dx \rho(x) \ln \rho(x) + (1 - \nu) \int dx \tilde{\rho}(x) \ln \tilde{\rho}(x) \\ & - \lambda \int dx dy \rho(x) \ln |x - y| \rho(y) - \nu \int dx dy \tilde{\rho}(x) \ln |x - y| \tilde{\rho}(y) \\ & - 2\kappa \int dx dy \rho(x) \ln |x - y| \tilde{\rho}(y). \end{aligned} \quad (13)$$

The hermitian Hamiltonian is finally given by

$$\begin{aligned}
H &\rightarrow J^{\frac{1}{2}} H J^{-\frac{1}{2}} \\
&= \frac{1}{2m_1} \int dx \rho(x) (\partial_x \pi(x))^2 + \frac{1}{2m_1} \int dx \rho(x) \left(\frac{\lambda-1}{2} \frac{\partial_x \rho}{\rho} + \lambda \oint \frac{dy \rho(y)}{x-y} + \kappa \oint \frac{dy \tilde{\rho}(y)}{x-y} \right)^2 \\
&+ \frac{1}{2m_2} \int dx \tilde{\rho}(x) (\partial_x \tilde{\pi}(x))^2 + \frac{1}{2m_2} \int dx \tilde{\rho}(x) \left(\frac{\nu-1}{2} \frac{\partial_x \tilde{\rho}}{\tilde{\rho}} + \nu \oint \frac{dy \tilde{\rho}(y)}{x-y} + \kappa \oint \frac{dy \rho(y)}{x-y} \right)^2 - \\
&\quad - \frac{\lambda}{2m_1} \int dx \rho(x) \partial_x \frac{P}{x-y} \Big|_{y=x} - \frac{\lambda-1}{4m_1} \int dx \partial_x^2 \delta(x-y) \Big|_{y=x} - \\
&\quad - \frac{\nu}{2m_2} \int dx \tilde{\rho}(x) \partial_x \frac{P}{x-y} \Big|_{y=x} - \frac{\nu-1}{4m_2} \int dx \partial_x^2 \delta(x-y) \Big|_{y=x}, \tag{14}
\end{aligned}$$

where P stands for the principal part. The two terms, quadratic in the conjugate momentum operators π and $\tilde{\pi}$, represent the kinetic energy of the system. The rest emerges as a quantum collective-field potential. The last terms are singular and do not give a contribution in the leading order in the $\frac{1}{N_1}$ and $\frac{1}{N_2}$ expansions. They should be cancelled by the infinite zero-point energy of the collective fields ρ and $\tilde{\rho}$.

To find the ground-state energy of our system, we assume that the corresponding densities are static. Since their momenta are vanishing, the leading part of the Hamiltonian in the $\frac{1}{N_1}$ and $\frac{1}{N_2}$ expansions is given by the effective potential

$$\begin{aligned}
V_{eff}(\rho, \tilde{\rho}) &= \frac{1}{2m_1} \int dx \rho(x) \left(\frac{\lambda-1}{2} \frac{\partial_x \rho}{\rho} + \lambda \oint \frac{dy \rho(y)}{x-y} + \kappa \oint \frac{dy \tilde{\rho}(y)}{x-y} \right)^2 \\
&+ \frac{1}{2m_2} \int dx \tilde{\rho}(x) \left(\frac{\nu-1}{2} \frac{\partial_x \tilde{\rho}}{\tilde{\rho}} + \nu \oint \frac{dy \tilde{\rho}(y)}{x-y} + \kappa \oint \frac{dy \rho(y)}{x-y} \right)^2. \tag{15}
\end{aligned}$$

Its form makes the Bogomol'nyi bound apparent. The potential is positive semi-definite and its contribution to the ground-state energy vanishes if there exist positive solutions of the coupled equations

$$\frac{\lambda-1}{2} \frac{\partial_x \rho}{\rho} + \lambda \oint \frac{dy \rho(y)}{x-y} + \kappa \oint \frac{dy \tilde{\rho}(y)}{x-y} = 0,$$

$$\frac{\nu - 1}{2} \frac{\partial_x \tilde{\rho}}{\tilde{\rho}} + \nu \oint \frac{dy \tilde{\rho}(y)}{x - y} + \kappa \oint \frac{dy \rho(y)}{x - y} = 0. \quad (16)$$

It is evident that there always exist uniform solutions

$$\rho(x) = \rho_0, \quad \tilde{\rho}(x) = \tilde{\rho}_0. \quad (17)$$

We have not been able to obtain analytic solutions to these equations for any values of the parameters λ, ν and κ . However, if we further simplify our model by the assumption that there are no three-body interactions between the particles in the starting Hamiltonian (1), we obtain the conditions (see Ref. [13])

$$\begin{aligned} \kappa^2 &= \lambda \nu, \\ \left(\frac{m_2}{m_1} \right)^2 &= \frac{\nu}{\lambda}. \end{aligned} \quad (18)$$

In this particular case, the solutions of equations (16) are always interrelated by

$$\tilde{\rho}^{\nu^{\frac{1}{2}} - \nu^{-\frac{1}{2}}} \sim \rho^{\lambda^{\frac{1}{2}} - \lambda^{-\frac{1}{2}}}. \quad (19)$$

Note that for $\lambda = \nu$, the condition (19) implies proportionality between ρ and $\tilde{\rho}$ and this means that we are dealing with the one-family model.

If we further assume that $\kappa = 1$ (weak-strong coupling duality in Ref. [19]), we end up with the condition

$$\rho \tilde{\rho} = c, \quad (20)$$

where c is some positive constant. This very condition allows us to find new soliton solutions to the coupled equations (16). In fact, there is only one relevant equation, let us say for ρ , for example

$$\frac{\lambda - 1}{2} \frac{\partial_x \rho}{\rho} + \lambda \oint \frac{dy \rho(y)}{x - y} + c \oint \frac{dy}{\rho(y)(x - y)} = 0. \quad (21)$$

This equation can be solved by a rational *Ansatz*

$$\rho(x) = \rho_0 \frac{x^2 + a^2}{x^2 + b^2}, \quad (22)$$

where a and b are positive constants. By using the Hilbert transform

$$\oint \frac{dy}{x-y} \frac{1}{y^2 + a^2} = \frac{\pi}{a} \frac{x}{x^2 + a^2}, \quad (23)$$

we find the conditions

$$\begin{aligned} \lambda - 1 + \frac{c\pi}{\rho_0 a} (b^2 - a^2) &= 0, \\ 1 - \lambda + \frac{\lambda \rho_0 \pi}{b} (a^2 - b^2) &= 0. \end{aligned} \quad (24)$$

The soliton solution for the first family is given by

$$\rho(x) = \rho_0 + \frac{\lambda - 1}{\lambda \pi} \frac{b}{x^2 + b^2}, \quad (25)$$

while the solution for the second family looks like

$$\tilde{\rho}(x) = \frac{c}{\rho(x)} = \frac{c}{\rho_0} + \frac{1 - \lambda}{\pi} \frac{a}{x^2 + a^2}. \quad (26)$$

We note that both solutions are localized at $x = 0$. For large values of x , the soliton solutions approach the uniform solutions found before. This yields one more condition

$$\frac{\rho(\infty)}{\tilde{\rho}(\infty)} = \frac{N_1}{N_2} = \frac{\rho_0^2}{c}, \quad (27)$$

which, together with the conditions (24), finally fixes the parameters a, b and c , namely

$$\begin{aligned} a &= \frac{N_1(\lambda - 1)}{\rho_0 \pi N_2 (1 - N_1^2 \lambda^2 / N_2^2)}, \\ b &= \frac{(1 - \lambda)}{\lambda \rho_0 \pi (1 - N_2^2 / \lambda^2 N_1^2)}, \\ c &= \rho_0^2 \frac{N_2}{N_1}. \end{aligned} \quad (28)$$

The particle number of the first soliton is

$$\int dx(\rho(x) - \rho_0) = \frac{\lambda - 1}{\lambda}, \quad (29)$$

while that of the second soliton is

$$\int dx(\tilde{\rho}(x) - \tilde{\rho}_0) = 1 - \lambda, \quad (30)$$

where $\tilde{\rho}_0 = \frac{c}{\rho_0}$. It is interesting to observe that these numbers are generally not integers.

For $\lambda < 1$, the first soliton behaves like the hole in the condensate ρ_0 and the second one behaves like the particle above the condensate $\tilde{\rho}_0$. The roles are interchanged for $\lambda > 1$. It is interesting to study the limiting case $c \rightarrow 0$. This can be achieved only if N_1 is much larger than N_2 . The condition (24) gives $a \rightarrow 0$ (the case $b \rightarrow \infty$ destroys the solution). The first-family soliton reduces to the "vortex" profile

$$\rho(x) = \rho_0 \frac{x^2}{x^2 + b^2}, \quad b = \frac{1 - \lambda}{\lambda \pi \rho_0}. \quad (31)$$

The above solution is called a vortex because the density goes to zero at $x = 0$. It has already been obtained in the collective-field approach to the one-family Calogero model [18, 20]. The second-family lump solution transforms into the sharp delta-function profile

$$\tilde{\rho}(x) = (1 - \lambda)\delta(x). \quad (32)$$

In deriving this result we have used the Lorentzian representation of a delta function

$$\delta(x) = \lim_{\varepsilon \rightarrow \infty} \frac{1}{\pi} \frac{\varepsilon}{x^2 + \varepsilon^2}. \quad (33)$$

We have thus demonstrated that our solitons have vanishing energy in the leading approximation. It does not cost any energy to create such a pair of correlated solitons.

What about the other solutions to the coupled equations (16)? The authors of Ref. [19] have claimed that there exists a multi-vortex solution in the first family accompanied by the sum of the delta-function profiles for the second family. The zeros of the $\rho(x)$ (i.e. the positions of the vortices) are simultaneously the points at which $\tilde{\rho}(x)$ diverges (i.e. the positions of the particles). According to Ref. [19], "the collective field $\tilde{\rho}(x)$ describes solitons as particles, whereas $\rho(x)$ gives a microscopic description in terms of fields". This is the essence of the so-called particle-vortex duality. We do not agree with this interpretation since $\rho(x)$ and $\tilde{\rho}(x)$ describe two different physical systems and, consequently, cannot be applied simultaneously to the same object (soliton in this particular case). Moreover, we doubt that there exist multi-vortex solutions at least not of the form suggested in Ref. [19]! Nevertheless, let us investigate this possibility more carefully in the case of the hypothetical two-vortex solution. This solution can be reached by the limiting procedure ($c \rightarrow 0$), starting from the two-hole solution in the condensate ρ_0

$$\rho(x) = \rho_0 \frac{(x^2 - a^2)(x^2 - \bar{a}^2)}{(x^2 - b^2)(x^2 - \bar{b}^2)}, \quad (34)$$

where a, \bar{a} and b, \bar{b} are complex parameters. This *Ansatz* for $\rho(x)$ takes into account the positiveness of the particle density automatically. Furthermore, the rational function of two polynomials with the fourth degree in x in principle ensures the two-hole (and for $\tilde{\rho} = \frac{c}{\rho}$, the two-lump) structure.

Inserting the expression (34) for ρ and $\frac{c}{\rho}$ for $\tilde{\rho}$ in equations (16), and by using the Hilbert transform

$$\oint \frac{dy}{x-y} \frac{1}{y^2 - a^2} = \pm \frac{i\pi x}{a(x^2 - a^2)}, \quad (35)$$

where the $+$ sign ($-$ sign) is taken for a in the upper (lower) half of the complex

plane, we obtain the following system of algebraic equations:

$$\begin{aligned}\lambda - 1 \pm \frac{c\pi i}{\rho_0 a}(a^2 - b^2)\frac{a^2 - \bar{b}^2}{a^2 - \bar{a}^2} &= 0, \\ 1 - \lambda \pm \frac{\lambda\rho_0\pi i}{b}(b^2 - a^2)\frac{b^2 - \bar{a}^2}{b^2 - \bar{b}^2} &= 0.\end{aligned}\tag{36}$$

Let us set

$$a = \alpha + i\beta, \quad b = \gamma + i\delta,\tag{37}$$

where α, β, γ and δ are real numbers. The real and the imaginary parts of the complex equations (36) are, respectively,

$$\begin{aligned}\lambda - 1 \pm \frac{c\pi}{\rho_0\beta}(\alpha^2 - \beta^2 - \gamma^2 + \delta^2) &= 0, \\ 1 - \lambda \pm \frac{\lambda\pi\rho_0}{\delta}(\gamma^2 - \delta^2 - \alpha^2 + \beta^2) &= 0, \\ 4\alpha^2\beta^2 - 4\gamma^2\delta^2 + (\alpha^2 - \beta^2 - \gamma^2 + \delta^2)(3\alpha^2 + \beta^2 + \gamma^2 - \delta^2) &= 0, \\ 4\gamma^2\delta^2 - 4\alpha^2\beta^2 + (\gamma^2 - \delta^2 - \alpha^2 + \beta^2)(3\gamma^2 + \delta^2 + \alpha^2 - \beta^2) &= 0.\end{aligned}\tag{38}$$

The only solutions are

$$\begin{aligned}\alpha = \gamma = 0, \quad \beta^2 &= \delta^2, \\ \alpha^2 = \gamma^2, \quad \beta = \delta &= 0, \\ \alpha = \beta = \gamma = \delta &= 0.\end{aligned}\tag{39}$$

This brings us back to the uniform solution $\rho(x) = \rho_0$. Similarly, one can show that the *Ansatz* $\rho(x) = \rho_0 \frac{(x^2 + a_1^2)(x^2 + a_2^2)}{(x^2 + b_1^2)(x^2 + b_2^2)}$, for real parameters a_1, b_1, a_2 and b_2 , leads to the one hole-lump pair solution, Eqs. (25) and (26).

In conclusion, the only duality that can be considered is the trivial self-duality of the collective Hamiltonian (14). Obviously, it is invariant under the interchange of the two families, namely $\rho \leftrightarrow \tilde{\rho}$, $\pi \leftrightarrow \tilde{\pi}$, $\lambda \leftrightarrow \nu$ and $m_1 \leftrightarrow m_2$.

Our final remark is that our two-family Calogero model without the three-body interaction (18) can be viewed as the one-family model, but only in the leading approximation. Namely, comparing the effective potential (15) with the effective potential for the one-family Calogero model, one finds that they can be identified. To this end, we can define the new, effective one-family collective field $\rho_{eff} = (\frac{m}{\mu^2 \lambda m_1})^{1/3}(\lambda \rho + \kappa \tilde{\rho})$, describing one-family particles with the effective mass m and effective one-family coupling strength μ . Note that this identification cannot fix the values of the afore-mentioned parameters. However, owing to the presence of the $\partial_x \rho / \rho$ and $\partial_x \tilde{\rho} / \tilde{\rho}$ terms, which are suppressed, respectively, by the factors $\frac{1}{N_1}$ and $\frac{1}{N_2}$ with respect to the leading terms, this equivalence breaks down. The derivative terms are crucial for the appearance of the soliton solutions. In this respect, the two-family models display the new, paired solitons which do not exist in the one-family Calogero models.

Although we have here focused on a two-family Calogero model, the discussion in this paper should be relevant to two- matrix models, since it is known that the Calogero model actually corresponds to the $O(N)$, $U(N)$ and $Sp(N)$ invariant matrix models for $\lambda = \frac{1}{2}, 1, 2$, respectively [21]. Our results can also be easily extended to the models with more than two distinct families of distinguishable particles. The open problems that still remain are the question of the quantum stability of the semi-classical solutions (25) and (26), and the existence of the possible moving-soliton solutions. We hope to study these issues in the near future.

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